

# The Position Estimation of AUV Based on Non-Linear Ensemble Kalman Filter Method

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**Abstract** – Monitoring of the condition of the unstructured and dangerous underwater requires an underwater vehicle that able to treat this condition. A vehicle that can be used for monitoring underwater is unmanned underwater vehicle, AUV (*Autonomous Underwater Vehicle*). AUV is robotic device that is driven through the water by a propulsion system, controlled and piloted by an onboard computer, and maneuverable in three dimensions. This research explains about position estimation of non-linear system of AUV based on The Ensemble Kalman Filter (EnKF). EnKF is used as the estimation method of AUV's position that maneuvering in 6 DOF (Degrees of Freedom) with the determined trajectory. The results show that the minimum error of estimation is 0.002 m and the maximum error is 0.123 m, and the higher ensembles need more the computational time.

**Keywords:** AUV, EnKF, Non-Linear System, Position Estimation

## 1. INTRODUCTION

Indonesia is a maritime country that has marine resources in large quantities. The resources include species of flora, fauna, microbes, coral reefs, renewable resources or nonrenewable resources and others. That resources need maintenance and monitoring regularly for safety of the country. Monitoring the condition of the unstructured and dangerous underwater needs an under water vehicle that can overcome the condition. Vehicle that can be used for that monitoring is unmanned underwater vehicle.

Unmanned underwater vehicle is being developed currently and it can be applied in several sector in life. That vehicle is important in many underwater activities because it has a high-speed, endurance and ability to dive more safely than humans [1]. One of the unmanned underwater vehicle is AUV (*Autonomous Underwater Vehicle*). AUV is a robotic device that driven through the water by a propulsion system, controlled and driven by the computer, and maneuverable in three dimensions [2]. AUV is important in underwater activities, so development of AUV should be done. One such development is trajectory estimation of AUV. The estimation requires an appropriate method, such as Data Assimilation. Data assimilation is an estimation method that combines mathematical models and measurement data [3].

One of data assimilation method is Kalman Filter. Kalman Filter is an estimation method for linear dynamic stochastic system. Some applications and modifications of Kalman Filter have been made to get a more accurate estimate and the computing time is shorter. One of the modifications of Kalman Filter is ensemble Kalman filter (EnKF). EnKF is an estimation method for non linear dynamic stochastic system based on measurement data. In this research, the authors develop a navigation and guidance control of AUV using EnKF. Dynamical model of AUV is 6 DOF (Degrees of Freedom). The position estimation is simulated with Matlab. Simulations show the estimation comparison and computation time between each of the number of ensembles.

## 2. METHODS

### 2.1 DYNAMICAL MODEL OF AUV MOTION

There are two coordinate system needed which are used to describe the motion of AUV: earth-fixed (inertial) coordinates and body-fixed coordinates. Earth-fixed coordinates are used to describe the position and orientation of AUV. That coordinates are the x-axis that pointing north, the y-axis that pointing east and the z-axis that pointing towards the center of the earth. Body-fixed coordinates are used to describe the velocity and acceleration of the vehicles. Its origin is usually set at the center of gravity or the center of buoyancy, the x-axis is positive towards the bow, the y-axis is positive towards starboard, and the z-axis is positive downward. AUV that used in this research is AUV SEGOROGENI ITS. AUV SEGOROGENI ITS uses only one propeller on the tail of AUV which will produce  $x_{prop}$  and additional moments  $K_{prop}$  [4].



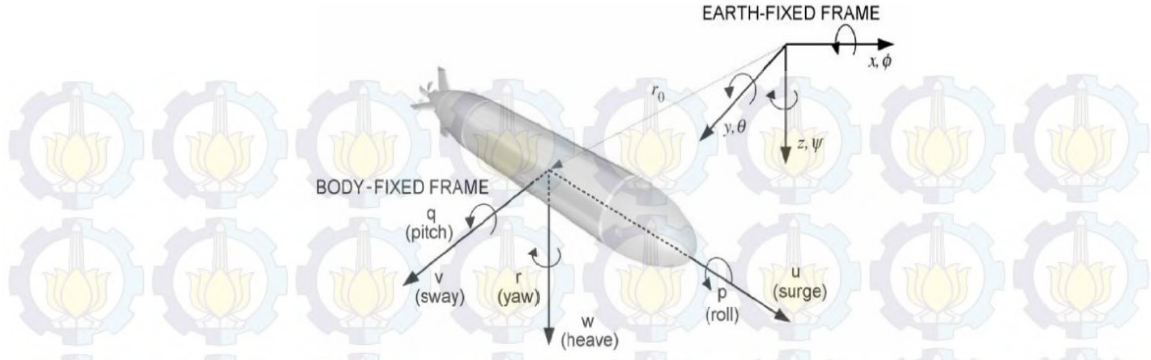


Figure 1. Coordinate System of 6 DOF AUV

The motion of AUV is 6 DOF, that is, three translations and three rotations along x, y, and z axis. The general 6 DOF (Degrees of Freedom) motion equations of AUV are non linear system. That equations consist surge, sway, heave, roll, pitch and yaw. That can be written as [5].

$$m[\ddot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] = X_{res} + X_{u|u}|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (1)$$

$$m[\ddot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{v|v}|v| + Y_{r|r}|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (2)$$

$$m[\ddot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{w|w}|w| + Z_{q|q}|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s \quad (3)$$

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p}|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (4)$$

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w}|w| + M_{q|q}|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (5)$$

$$I_z\dot{r} + (I_y - I_x)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v}|v| + N_{r|r}|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + M_{uu\delta_r}u^2\delta_r \quad (6)$$

We need to transform linear and angular velocity of AUV to be position and orientation. The following equations define that transformation[6].

$$\dot{x} = u \cos(\psi) - v \sin(\psi) \quad (7)$$

$$\dot{y} = u \sin(\psi) + v \cos(\psi) \quad (8)$$

$$\dot{z} = w \quad (9)$$

$$\dot{\psi} = r \quad (10)$$

## 2.2 ENSEMBLE KALMAN FILTER

The model of AUV system is not exactly same with the real system, there is noise system that can not be written in the model. So that can be written as

$$x_{k+1} = f(x_k, k) + w_k \quad (11)$$

With  $x_{k+1}$  is the state variable in time  $k + 1$ , the velocity of AUV.  $w_k$  is noise system, which is a random vector drawn from Gaussian distribution with mean = 0 dan covariance  $Q$ .

An observation equation is defined to make correlation between the state which we estimate and the observation model. The equation as follow:

$$z_k = Hx_k + v_k \quad (12)$$

With  $z_k$  is the observation.  $H$  is a matrix representing the observation model and  $v_k$  is the observation noise, which is a random vector drawn from Gaussian distribution with mean = 0 and covariance  $R$ .

Suppose, we have a dynamic stochastic system Eq. (11) and observation Eq. (12). By using Ensemble Kalman Filter method we estimate the state variables of Eq. (1-6) with the observation data Eq. (12). Ensemble Kalman Filter is one of the modifications of Kalman Filter method. Kalman Filter is the estimation method for linear dynamic stochastic



system, meanwhile Ensemble Kalman Filter is the estimation method for non linear dynamic stochastic system based on the measurement (observation) data.

The algorithm of the Ensemble Kalman Filter is [3]

a. The Initial Estimation

Generate the n-ensembles of initial estimation  $x_{0,i} = [x_{0,1} \ x_{0,2} \ x_{0,3} \ \dots \ x_{0,n}]$  with  $x_{0,i} \sim N(\bar{x}_0, P_0)$ .

Mean of intial estimation which generated :  $\hat{x}_0 = \frac{1}{n} \sum_{i=1}^N \hat{x}_{0,i}$

b. The Prediction Step

$$\hat{x}_{k,i}^- = f(\hat{x}_{k-1,i}, u_{k-1}) + w_{k,i} \quad (13)$$

with  $w_{k,i} \sim N(0, Q_k)$  is the ensemble of noise system. Mean of prediction step estimation :  $\hat{x}_k^- = \frac{1}{n} \sum_{i=1}^N \hat{x}_{k,i}^-$

Error covariance of prediction step estimation :  $P_k^- = \frac{1}{n-1} \sum_{i=1}^N (\hat{x}_{k,i}^- - \hat{x}_k^-) (\hat{x}_{k,i}^- - \hat{x}_k^-)^T$

c. The Correction Step

Generate the ensemble of measurement data

$$z_{k,i} = z_k + v_{k,i} \quad (14)$$

with  $v_{k,i} \sim N(0, R_k)$  is the ensemble of measurement noise. Kalman gain is defined as  $K_k = P_k^- H^T (HP_k^- + R_k)^{-1}$

Estimation of correction step is

$$\hat{x}_{k,i} = \hat{x}_{k,i}^- + K_k(z_{k,i} - H\hat{x}_{k,i}^-) \quad (15)$$

Mean of correction step estimation  $\hat{x}_k = \frac{1}{n} \sum_{i=1}^N \hat{x}_{k,i}$  with error covariance is  $P_k = [1 - K_k H] P_k^-$ .

d. Substitute Eq. (15) in prediction step Eq. (13)

e. Repeat and continue step (b) and step (c) until we get mean of correction step estimation as the result of estimation.

### 3. RESULTS AND DISCUSSION

#### 3.1 MODEL DISCRETIZATION

The state space is derived from the model of AUV motion Eq. (1-6). The State space is used in the discretization step. The step of that as follows

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m-X_u} & -\frac{my_G}{m-X_u} \\ 0 & 1 & 0 & -\frac{mz_G}{m-Y_v} & 0 & \frac{(mx_G-Y_v)}{m-Y_v} \\ 0 & 0 & 1 & \frac{my_G}{m-Y_v} & -\frac{mz_G}{m-Y_v} & 0 \\ 0 & -\frac{mz_G}{I_x-K\dot{p}} & \frac{my_G}{I_x-K\dot{p}} & \frac{mz_G}{m-Z\dot{w}} & -\frac{mz_G}{m-Z\dot{w}} & 0 \\ \frac{mz_G}{I_y-M\dot{q}} & 0 & -\frac{(M\dot{w}+x_G)}{I_y-M\dot{q}} & 1 & 0 & 0 \\ -\frac{my_G}{I_z-N\dot{r}} & \frac{(mx_G-N\dot{r})}{I_z-N\dot{r}} & \frac{I_y-M\dot{q}}{I_z-N\dot{r}} & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \\ B6 \end{pmatrix} \quad (16)$$

with B1, B2, B3, B4, B5 and B6 are non linear equation from the dynamical model of motion AUV.

The general form of Eq. (16) can be written as

$$A\dot{x} = B \quad (17)$$

So the model system of AUV motion as follows

$$\dot{x} = A^{-1}B \quad (18)$$

Before we applied the Ensemble Kalman Filter to estimate the position of AUV, we discretize Eq. (18) respect to time,  $t$ , by using the Forward Finite Difference Method.

$$\dot{x} = \frac{x_{k+1} - x_k}{\Delta t} \quad (19)$$

The state space form of model of AUV motion is

$$x_{k+1} = f(x_k, k) \quad (20)$$

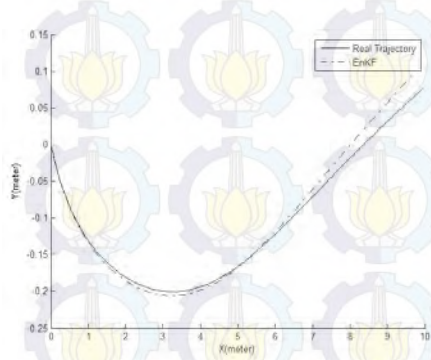
#### 3.2 SIMULATION

Simulations in this research are in 2 dimension of graph. The graphs show the position of AUV in x, y and z axis based on model of AUV motion 6 DOF. The real number of estimation is determined trajectory. There are three trajectories: straight trajectory, turning trajectory and diving trajectory. The measurement systems of each cases are surge, sway, heave, and yaw with initial angle is  $\delta_r = 5^\circ$  and the change of time is  $\Delta t = 0.001$ . The initial position of trajectories are  $x(0) = 0$  m,  $y(0) = 0$  m and  $z(0) = 0$  m with initial velocities are  $u(0) = 0$  m/s,  $v(0) = 0$  m/s,  $w(0) = 0$  m/s,  $\phi(0) = 0$  m/s,  $\theta(0) = 0$  m/s, and  $\psi(0) = 0$  m/s. The results of estimation are derived from the average of 10 times estimation. The following simulation shows the result of estimation with 100 ensemble in figure 2.

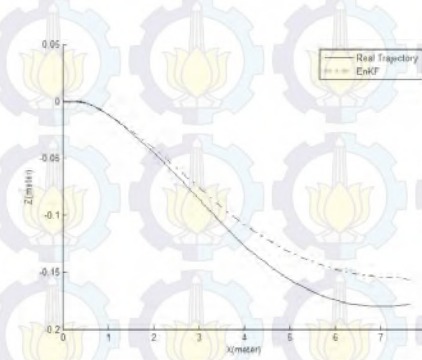
The accuracy of those estimation is represented by the value of RMSE. RMSE is derived from the average of 10 times of the simulation and represented bellow in table 1. RMSE is divided in to 4 ensembles, they are 50, 100, 200 and 300 ensembles. The minimum value of RMSE is blocked.



a. Case I : Turning trajectory in xy-position



b. Case II : Diving trajectory in xz-position



c. Case III : Straight trajectory in xy-position

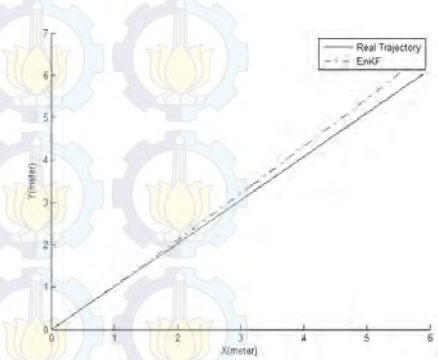


Figure 2. The Position Estimation of AUV in Three Trajectories

Table 1. The Error of The Position Estimation of AUV

Ensemble	Trajectory 1			Trajectory 2			Trajectory 3		
	RMSE			RMSE			RMSE		
	X (m)	Y (m)	Angle (rad)	X (m)	Z (m)	Angle (rad)	X (m)	Y (m)	Angle (rad)
50	0,03878	0,00836	0,002008	0,05852	0,01534	0,04375	0,04283	0,12648	0,010039
100	0,03827	0,00823	0,002008	0,05553	0,01524	0,04353	0,04218	0,12419	0,010039
200	0,03786	0,00820	0,002008	0,05572	0,01521	0,04344	0,04173	0,12387	0,010039
300	0,03780	0,00816	0,002008	0,05627	0,01516	0,04341	0,04133	0,12342	0,010039

The higher ensembles need more computational time. The computational time of the simulation is bellow in table 2.

Table 2. The CPU Time of Algorithm

Ensemble	Computational Time (s)		
	Trajectory 1	Trajectory 2	Trajectory 3
50	4,17	1,70	3,32
100	7,10	3,42	6,74
200	13,90	6,77	13,40
300	21,98	9,51	19,00

## 4. CONCLUSIONS

From description and the result of estimation we conclude that:

We can apply the Ensemble Kalman Filter to estimate the position of AUV based on determined trajectories. And the higher ensembles need more the computational time. For the next research, we can take controller in estimation and also we can try for another measurement systems.

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